A MODIFIED WEIBULL ANALYSIS FOR FRACTURE TEST DATA OF TABLETS WHICH VARY IN VOLUME AND DENSITY

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The inevitable flaws in brittle materials give rise to a wide variation in the mechanical strength of nominally identical test specimens. This variability can be characterised by statistical techniques; the analysis most widely adopted for this purpose is that proposed by Weibull (1939). There is evidence (STANLEY and NEWTON, 1978) that the Weibull-based statistical model satisfactorily represents the strength variability of nominally identical right circular cylindrical powder compacts, as measured by the diametral compression test. However, powder compacts manufactured on conventional production machines exhibit inconsistences in their weights, thicknesses and forming pressures in addition to the strength variability inherent in the material. These additional forms of variability will give rise to an increase in the observed strength variability of such compacts over and above that attributable to the material itself.

The Weibull analysis has been modified so that an estimate of the intrinsic material strength variability can be made from non-identical powder compacts. The resulting modified Weibull analysis is applicable to powder compacts exhibiting variability in two parameters, in addition to intrinsic strength. After detailed consideration the two parameters chosen for this study were compact volume and compact density.

The modified equation used for the calculation of the probability of failure, P_f associated with a fracture stress, σ_f, took the form:-

$$P_f = \int_0^{\infty}\int_0^{\infty} f_{(\rho v)} \left\{1 - \exp\left[-\left(\frac{1}{m\,!}\right)^m \left(\frac{\sigma_f}{\bar{\sigma}_{(\rho v)}}\right)^m\right]\right\} dv d\rho$$

where $f_{(\rho v)}$ is the relative frequency of occurrence of compacts, within the test batch, with volumes and densities in the ranges v to v+dv and ρ to $\rho+d\rho$ respectively,

m the Weibull modulus, is a reciprocal measure of the strength variability,

$\frac{1}{m\,!}$ is the "gamma" function of $\frac{1}{m} + 1$,

$\bar{\sigma}_{(\rho v)}$ is the mean fracture stress of compacts of a specified volume v and density ρ.

Using this equation experimental data can be processed so that the intrinsic material strength variability can be isolated from that due to random variations in volume and density. Fracture test results for a batch of 1005 aspirin tablets, produced on an instrumented Manesty E2 single punch tablet machine, have been analysed. The numerical evaluation of the integrals in the equation required the construction of a bivariate histogram to represent the distribution of compact volume and compact density. The unmodified Weibull modulus was calculated to be 12.23 whereas the modified Weibull modulus was 15.16; this latter value compares well with a reported value of 15.54 for 31 identical tablets selected from the entire batch (KENNERLEY, NEWTON and STANLEY, 1979) and reflects the material strength variability rather than that of the sample.

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